Physics 101 Course Notes #6

Motion and Interaction of Particles

Concepts: Velocity and acceleration

Newtonian Theory of Mechanics

Central goal of mechanics: To invent a theory capable of explaining and predicting how objects move in a wide range of circumstances.

Interaction: An object A interacts with another object B if A is affected by the presence of B.

"Is there a relation between motion and interaction?"

Our approach : From simple → complex situations

Focus on particles (an object whose position can be adequately specified by that of a point). Later move from a few particles to many.

Single Non-interacting Particle

Hypothesis: "Every single particle moves relative to the stars with constant velocity"

Motion relative to other reference frames

P: particle, S: star, R: rocket

Inertia Principle: "There exists reference frames relative to which every non-interacting particle moves with constant velocity. Any such frame is called an **INERTIAL FRAME**"

-Rocket moving with constant velocity?
- Earth rotating?
- A falling down elevator?
- Distant Stars?

Two Interacting Particles

-far apart →?

-sufficiently close →?

- Examples?

" The interaction of a particle depends on the properties of both interacting particles"

Position Dependent Interactions

Assumption: "The interaction between two particles at any distant depends only on their relative positions at that instant"

"The acceleration of a particle, at any instant depends only on the distance between the two interacting particles and is directed along the line joining them."

Repulsive:

Attractive:

Long-range interaction: Gradual change

Contact interaction: atomic size closeness, touching, nontouching

Reciprocity of interaction

-identical particles

-directions

General case:

"When two particles interact, their accelerations at any distant are oppositely directed. The ratio of their magnitudes is a constant depending only on the nature of these particles"

Two particles: Mass and Force

Mass:

-similar to time and length (comparative definition)

Mass ratio

smaller acceleration $\rightarrow$ larger mass

- A property describing the particle's tendency to maintain its velocity

- Inertia

-Unit: SI, kg.

Relation between accelerations:

FORCE:

Defn.: " The force on a particle by other objects is a quantity which depends on the properties of all the interacting objects and which is equal to $m\vec{a}$ where m is the mass of the particle and $\vec{a}$ its acceleration."

-Mutual forces have equal magnitudes and opposite directions
-Unit?

Central Force: "At any instant, the force on a particle by another depends only on the distance between them, and is directed along the line joining them."

Three or more interacting particles
Superposition principle

Any number of particles

Newton's Law:

-Why important?

Newton's Laws of Mechanics

Law #1: There exists reference frames (called inertial frames) relative to which every non-interacting particle moves with constant velocity
Law #2: The acceleration $\vec{a}$ relative to an inertial frame of any particle of mass m is given by
Law # 3: The interaction between any two particles 1 & 2 can be described by a pair of mutual forces having the following properties
a)
b) depends only on the distance between the particles and is directed along the line joining them.

- particles only complex systems
- tremendous range of applications in science and engineering
- limitations?

Exploiting Newton's Laws

$$m\vec{a} = \vec{F}_{tot}$$

-generality: any particle,

-specific particle

- component form

Composite particles

- All parts move with same velocity and same acceleration $\rightarrow$ composite particle

Common Interactions

Long-range interactions

Gravitation:

-If a particle interacts only with earth: ?

Weight: magnitude of gravitational force

W = mg

Contact Interactions:

-Atoms

-touching (10^{-10} m)

Interaction with a spring

Direction: Along the spring, opposing deformation

Magnitude: Increasing with deformation, zero if no deformation

Interaction with a string

-Rubber band: Oppose elongation, does not oppose compression, lacks rigidity

- String: like rubber band, only small elongation, length remains constant

Direction: Attractive (opposing elongation)

Magnitude: Non-zero if the string is stretched, zero if not

Strings of negligible mass

-forces acting on its ends are equal

-tension same at every point of a string (provided that no other forces parallel to the string act on it)

-Also valid for curved strings

Interaction between non-adhering touching objects

-two touching objects

- interaction between the atoms near the surface of contact

Normal Force:

-Oppose compression $\rightarrow$ repulsive

Direction: perpendicular to the contact surface, repulsive (opposing compression)

Magnitude: $\neq 0$ if touching, $=0$ otherwise

Friction force: Parallel to surface, opposing relative sliding

Direction:?

1) Determine how the objects will move if $f = 0$

2) f will be in reverse direction to this motion

Kinetic case: (Objects moving relative to each other)

-f is directed opposite to relative velocity

Static case: (Objects at rest relative to each other)

- friction force is such that relative acceleration = 0

Problem Solving in Mechanics

Systematic application of Newton's Law

$$\mathbf{m\vec{a}} = \mathbf{\vec{F}_{tot}}$$

-express correctly and conveniently

1) Describe all ingredients

2) Use this description to express Newton's Laws

System Description by a System Diagram

System: Any object, or set of objects which one wishes to consider

System Diagram: (free-body diagram)

Construction:

1) Separate system

2) Mass

3) Motion (velocity and acceleration)

4) All forces on the system

    -Long range (e.g. gravity): interacting objects, forces on the systems

    -Contact:

        * Touching objects? (Mark and label)

        * Forces on the system

5) Components:

Example: Sled Sliding down along a hill

Related System Diagrams
-Interacting systems
-Consider separately
- If mutual interaction → opposite mutual forces with same magnitude
Example: Pulling a sled carrying a box

Applying Newton's Law in component form

$\mathbf{m\vec{a}} = \mathbf{\vec{F}_{tot}}$ any direction

(mass)x(acceleration component along this direction) = sum of all forces along this direction

PROBLEM SOLVING METHOD

Analysis of a problem
-Basic Description
    * Situation

-Known Information
-Diagram and useful symbols
* Goals
-Refined Description
* Time sequence and intervals
* Physics Description (motion, interaction)

Construction of Solution

Choose a subproblem and implement

Subproblem options

Find useful relation: Appl Newton's Law to system, at time, along direction

Checks:
- Goals attained?
- Well Specified?
- Self consistent?
- Other consistent?
- Optimal?

Example: Banking of a road curve

Further Suggestions:

- Start from Newton's Law

- Introduce auxiliary unknown quantities

- Solutions justified by basic physics principles?

- Work with symbols

More about interactions

Spring Force for small deformations

-if |x| is sufficiently small

$F_x = -kx$

k: spring constant

Friction Force:

Kinetic Friction:

$f = \mu_k N$

Direction opposite to relative velocity

Static friction:

$f \leq \mu_k N$

Direction so that relative acceleration = 0

f so that relative acceleration = 0

Gravitational Force:

-Spherical Body

Applications: Planetary Motion

Electric Force:

Electric Interactions

-electrons, protons carry electric charge

-can be negative and positive

Magnitude:

direction: Repulsive if charges have same signs

Attractive id they have opposite signs

Predictive power of Mechanics

$$m\vec{a} = \vec{F}_{tot}$$

- A knowledge of position and velocities of particles at any time allows one to predict their positions and velocities at any other time: IMPLICATIONS?

Physics 101 Course Notes #7

KINETIC ENERGY AND WORK

-How can we use Newton's Law to find v(r) ?

Kinetic Energy Law for small displacements

Kinetic Energy:	K = 1/2mv2
Infinitesimal Work:	$d'W = Fdr_F$
Kinetic Energy Law:	$dK = d'W$

-New concepts

Kinetic Energy

- $K=mv^2/2$ number (not a vector)
- units kg $(m/s)^2 = kgm^2/s^2 \equiv$ Joule

Infinitesimal Work

-describes interaction between particle and object

1) Identify the magnitude F
2) Evaluate Fdr_F (dr_F can be negative or positive)

- d'W can be negative or positive

-Units: Nm = $kg(m/s^2)m = kgm^2/s^2$ = Joule

General Kinetic Energy Law

$\Delta K = W_{tot}$	**Kinetic Energy Law**

Work done by some common forces

Constant Force:

-Path Independence: The work done by a constant force is independent of the path.

Friction Force

-moving particle

- f constant, opposite to particle's displacement

-work done by friction force depends on the path

Work and Vector Dot Product

Superposition Principle

$W = \Sigma W_i$

- Numerical sum not a vector sum

Utility of Superposition Principle

1) Calculate total force by adding, then calculate total work

2) Calculate W_i separately, then add all these works

Example: Speed of a pendulum

Power

-Assume d'W is done in dt

-rate of doing work

Power P = d'W/dt

-if the work done in a shorter time, power is larger

-Units: Joule/sec ≡ Watt (kg m2/s3)

Power delivered to a moving particle

Potential Energy and the Energy Law

Conservative Force: A force which does the same work independent of the path

Standard Definition for Calculating Work

-work done between A and B can be found by simple subtraction

Potential Energy $U_A = W_{AS}$

Defn: The potential energy UA (due to specified conservative forces) of a particle at a point A, relative to some standard position S is the work done on the particle by these forces along any path from A to S.

Relation between work and potential energy

$W = -\Delta U$

Gravitational Potential Energy near the earth

$U = mgh$

Energy Law

$\Delta E = W_{oth}$

Properties of the Energy

- hybrid quantity describing motion and interaction
- unit = Joule
- can be either negative or positive
- direct relation between a particle's speed and position

Comparison with Kinetic Energy Law

Conservation of Energy

Problem Solving with the Energy Law

Available Mechanics Laws

- Newton's Law, $m\vec{a} = \vec{F}_{tot}$

  * Relation between motion and interaction at any instant

  * how motion varies in the course of time

-Energy Law, $\Delta E = W_{oth}$

  * Motion and Interaction

  * Relation between particle's speed and position at any two instants, without mention of the elapsed time

Find Useful Relation

- Apply the Newton's Law to a particle at time, along direction

- Apply the Energy Law to a particle between times

Conservative property of Central Forces

Central Forces: direction along the line joining two particles, magnitude: depends only on the distance

-gravitational force, electric force

- work done by central force is independent of path

- Central forces, by one or more particles, are conservative

Gravitational and Electric Potential Energies

- Calculation of the potential energy

Potential Energy of Spring Forces

$U = kx^2/2$

Grand Summary of Mechanics Laws

Newton's Law, $m\vec{a} = \vec{F}_{tot}$

Validity: For inertial frame

Utility: Relates motion and interactions at any time

Energy Law, $\Delta E = W_{oth}$

Validity: For inertial frame

Utility: Relates speeds and positions at any two instants (without mention of time)

Physics 101 Course Notes #8

MOTION and INTERACTION OF SYSTEMS

Momentum

- How can we apply Newton's Law to systems consisting of several particles?

SYSTEM: Any set of particles selected for consideration

Fint: The sum of forces on all particles in the system by all particles inside the system
Fext: The sum of forces on all particles in the system by all particles outside the system.

-Momentum of a particle
-Newton's Law in terms of momentum
-Momentum of a system
-Momentum Law

1) Newton's Law is a special case of momentum law for one particle
2) Relation between motion and interactions
3) Apply momentum law to **system** at **time** along **direction**

Choice of a system: Choose a system so that unknown or uninteresting forces are internal, and so that forces of interest are external.

CONSERVATION OF MOMENTUM

-also valid for components

Momentum change in short collisions

Short Collisions:

1) Momentum can change appreciably due to internal forces
2) position change minimal
Problems involving collisions
1) Use conservation of momentum to relate the velocities of particles immediately before and after their collision
2) Use mechanics laws (Newton and Energy) to find information about the particles' motion during the times before and after the collision.

Motion of the Center of Mass

-Position of center of mass
-Location of the center of mass
-simple properties of center of mass
-**Symmetric objects:** If an object has a plane of symmetry, its center of mass must be located on this plane
-Composite objects

Energy of a System

-previously for particles, now for systems

$\Delta E = W_{oth}$ GENERAL ENERGY LAW

Internal Work and Potential Energy
-internal force on a system is zero
-However, internal work and internal potential ordinarily is not zero.

Mutual work on a pair of particles
1) Mutual work depends only on their displacement relative to each other
2) Mutual potential energy also depends on their positions relative to each other
3) U_{int} is just the sum of potential energies between **all pairs** of particles in the system

System with Constant Interparticle distances
-Rigid Bodies (metal plates, rocks), String, Liquid (water incompressible)
-If interparticle distances are constant $\Delta U_{int} = 0$

Systems Energies and center of Mass
-Kinetic Energy

Gravitational Potential energy near the Earth
-as if the system's entire mass were concentrated at its center of mass

Energy of Atomic Systems
Conservation of Energy: Forces between atomic particles are conservative
E=constant

Energy Conservation in Collisions
-Collision between Elementary Particles
-Collision between complex particles

Elastic Collision: A collision where the kinetic energies associated with the motions of center of mass remains constant
-Collision between elementary particles are ELASTIC

Inelastic collision: A collision where the Kinetic Energy does not remain constant
-Collision between complex particles maybe both elastic or inelastic

Energy of Macroscopic Systems
Macroscopic systems: Systems with large number of atoms
-10^{-10} m, atom, 10^{24} atoms per cm3
-macroscopic properties

Atomistic: Consider every particle. Infer general properties of macroscopic systems

Energy law for a Macroscopic System

Macroscopic and Non-Macroscopic Energies
$E = E_{mac}+E_{nmac}$

E_{mac}: Kinetic Energy of Center of Mass, potential energy due to external forces (e.g. gravity)
E_{nmac}: $E_{nmac} = E - E_{mac}$

Thermal Energy: E_{th}: Energy associated with the random motion of molecules
Gas: Just kinetic energy
Liquid and Solid: K.E. + Pot. Energy due to the interaction among molecules

Chemical Energy: Potential Energy due to interaction within molecules
$E_{nmac} = E_{th} + E_{chem}$
Energy Transfers and Transformations
-system in thermal constant, isolated
-Heat:

Most Random Distribution of Energy
-Thermal Energy Transfer will continue until the average thermal energy of each molecule becomes the same
-Transfer from warmer to colder system
-Temperature is closely related to average thermal energy of a molecule
-Transfer will continue until T=T'
Examples for Transformation of Macroscopic Energy to Thermal Energy
1) Pendulum
2) Sliding Crate coming to a rest
3) Ball bouncing on the floor
4) Bullet hitting a wall

Irreversibility of Energy Transformation

Examples of Complex Energy Transformation
-Jumping
-Jump Height and animal size

Friction and Energy Dissipation
Friction Force: A convenient way for a macroscopic description of systems

GRAND SUMMARY: Mechanics Laws for any system
Momentum Law
-Relates motion and external Interactions
-Relates velocities at any two instants (if $F_{ext} = 0$)

Energy Law
-Relates speeds and positions at any two instants (without mention of time)

Physics 101 Course Notes #9

ROTATIONAL MOTION

Angular Description of Motion
- Difference points move different amounts
- New ton's law focus on **linear** motion

-describe rotational motion in terms of angles and also express mechanics laws in terms of angles

Angular Displacement: A quantity specified by a rotation axis, an angle of rotation, and a sense of rotation

Angular Position: An object's angular displacement relative to some standard orientation

-Units:

Rates of Change of Rotational Quantities

-Angular Velocity

-Angular acceleration

Relating Angular Displacement and Their Rates of Change

Relations Between Angular and Linear Motion

-Rotational Kinetic Energy

-Moment of Inertia: A relation between an object's rotational kinetic energy and its angular velocity

-Moment of Inertias of dumbbell, Ring, Rod, Rectangular Plate, Disk,

Moments of Inertia About Different Axes

-Parallel Axes Relation

-Moments of Inertia about perpendicular axes

Rotational Torque

-involves force and distance

-describes how interaction effects rotation

-torque on **particle** by **force** about **axis**

-sign: negative or positive

-Units: N.m

Rotational WORK

Work on a system: Sum of works done by all the torques on a system

ANGULAR MOMENTUM LAW

-Describes the motion of a particle by its angular momentum, and its interaction by the total torque on it

-Relation between motion and interaction

-Valid even if the particle's distance, r, from the rotation axis changes

Alternative expressions

-Angular momentum closely related to linear momentum

-Units: $kg.m/s^2$

Angular Momentum Law for a system

-relation between motion of system and its interactions with other systems

-especially useful for rotational motion

-Rigid Object

-Gravitational Torque

Analogies between linear and rotational motions

Problem Solving and Examples

Conservation of Angular Momentum
-Rotation with constant moment of inertia
-Rotation with changing moment of inertia
-Short Collisions

GRAND SUMMARY
-Momentum Law
-Angular Momentum Law
-Energy Law

ROLLING and EQUILIBRIUM

Rolling: Rigid Objects
Translation: Can be described by the motion of center of mass

Rotation about a moving axis
-rigid object moving in a plane
-can be divided into two successive operations: 1) Translation, 2) Rotation
- Description in terms of center of mass

ROLLING MOTION
-rolling without slipping
-no slipping, contact point does not move
-friction force does no work on the wheel
-no transformation of macroscopic energy into random thermal energy
-very efficient means of transportation

Acceleration of a rolling cylinder

Speed of a rolling cylinder

STATIC EQUILIBRIUM
Defn: A system is said to be in equilibrium relative to some reference frame if it remains at rest relative this frame

Conditions
-Sufficient information for rigid objects
-angular momentum law condition can be applied about any convenient axis

Example: Ladder leaning against a wall

Physics 101 Course Notes #10

OSCILLATIONS
Defn: Periodic motion of an object around an equilibrium point.
-Simple Harmonic motion
Energy Considerations

-Energy is proportional to the square of the amplitude
Applications
-Pendulum
-